

Your challenge to calculate PI by hand

This is an excerpt from my other file at "[engert.us/erwin/miscellaneous/William Shanks 707 digits.pdf](http://engert.us/erwin/miscellaneous/William%20Shanks%20707%20digits.pdf)". Here is how you can calculate PI without a calculator or a computer. This is one of the most used formula is $PI = 4 * (4 * ATN(1/5) - ATN(1/239))$. The function $ATN(X) = X - X^3/3 + X^5/5 - X^7/7 + X^9/9$ etc. can be used to calculate the ATN function. The value for $X^3/3$ is the same as $X*X*X / 3$. The process continues until the value reaches enough 0's and no longer affects the total sum. See the next page how it works to produce the values needed.

If you would like to see if you are able to do a small sample of what William Shanks did, but he did on a much larger scale. To do it correctly all the work must be done by one person with paper and pencil only, no calculating type device as a calculator or computer even an abacus. This would require you to know how to do long division without any aid other than your own mind plus paper and pencil. Can you do the challenge or will you just say it is too hard I am not capable to do the task, which are you. Without looking at the following page except to see if you can do the first two terms correct. The work can be done in several days if you work hard and steady and long, or use a group effort to speed up the work. There are some short cuts starting with doing the powers of 1/5, after the first term divide by 25 from sub term to sub term. In this case multiply by 4 and divide by 100 which are the same as shifting to the right by two digits. Then do the division by the term number i.e. 3, 5, 7 etc. The value of 1/5 is .2 while the next term is .2 times 4 is .8 then shift to the right by two digits which is .008, which then has to be divided by 3 which gives you .00266666 etc.

These formulas are sometime referred to as Machin-like formula. There are many longer formulas that take more total terms to reach the same number of digits. Remember to alternate the addition and subtraction of every other term. Have fun with this problem. If you want you could use any of the following formulas listed on page 18 with further detailed information.

An early calculation not mentioned as often also had the same error as his 1873 work. The total term listing for 530 digits with 607 digits for PI is found in William Shanks 1853 calculation book "Contributions to mathematics, comprising chiefly the rectification of the circle". If someone would like to figure out where the error is in which term has the error. The next entry I have supplied term level for PI to 720 digits. One error was found to be in the term $(1/5)^{497}/497$ he omitted a 0 in the term in the 531 digit he had 8482897 instead of 80482897, his 1853 book just missed displaying this error. This 0 would repeat at digit 741. This error only corrects an extra 38 digits, $1/497$ has a repeat pattern of 210 and the $(1/5)^{497}$ fills more than the 707 digit position, the repeat starts at 708 digits. In the first series there still are more errors, and there is still an error in the second series. Due to William Shanks error all new records have to be calculated using two different methods, so his work was not in vain, even in 1947 they knew they needed to do two calculations. If you would like to see the detailed listing of each term William Shanks went through see the following web link "engert.us/erwin/miscellaneous/PI.pdf".

Here is a term by term breakdown for the solution of computing PI to 35 digits. Note how each term is getting closer to 0. You must always calculate more digits than required to for the accuracy you desirer hence 40 digits to get a 35 digit answer. The reason I picked 35 digits was that Ludolph Van Ceulen (1540 - 1610) spent most of his life working out PI to 35 decimal places. PI is sometimes known as Ludolph's Constant the value is carved on his tombstone.

(1/5)=	+	0.2000000000	0000000000	0000000000	0000000000
(1/5)^ 3/3=	-	0.0026666666	6666666666	6666666666	6666666666
(1/5)^ 5/5=	+	0.0000640000	0000000000	0000000000	0000000000
(1/5)^ 7/7=	-	0.0000018285	7142857142	8571428571	4285714285
(1/5)^ 9/9=	+	0.0000000568	8888888888	8888888888	8888888888
(1/5)^ 11/11=	-	0.0000000018	6181818181	8181818181	8181818181
(1/5)^ 13/13=	+	0.0000000000	6301538461	5384615384	6153846153
(1/5)^ 15/15=	-	0.0000000000	0218453333	3333333333	3333333333
(1/5)^ 17/17=	+	0.0000000000	0007710117	6470588235	2941176470
(1/5)^ 19/19=	-	0.0000000000	0000275941	0526315789	4736842105
(1/5)^ 21/21=	+	0.0000000000	0000009986	4380952380	9523809523
(1/5)^ 23/23=	-	0.0000000000	0000000364	7220869565	2173913043
(1/5)^ 25/25=	+	0.0000000000	0000000013	4217728000	0000000000
(1/5)^ 27/27=	-	0.0000000000	0000000000	4971026962	9629629629
(1/5)^ 29/29=	+	0.0000000000	0000000000	0185127900	6896551724
(1/5)^ 31/31=	-	0.0000000000	0000000000	0006927366	6064516129
(1/5)^ 33/33=	+	0.0000000000	0000000000	0000260301	0482424242
(1/5)^ 35/35=	-	0.0000000000	0000000000	0000009817	0681051428
(1/5)^ 37/37=	+	0.0000000000	0000000000	0000000371	4566310054
(1/5)^ 39/39=	-	0.0000000000	0000000000	0000000014	0963029202
(1/5)^ 41/41=	+	0.0000000000	0000000000	0000000000	5363471355
(1/5)^ 43/43=	-	0.0000000000	0000000000	0000000000	0204560302
(1/5)^ 45/45=	+	0.0000000000	0000000000	0000000000	0007818749
(1/5)^ 47/47=	-	0.0000000000	0000000000	0000000000	0000299441
(1/5)^ 49/49=	+	0.0000000000	0000000000	0000000000	0000011488
(1/5)^ 51/51=	-	0.0000000000	0000000000	0000000000	0000000441
(1/5)^ 53/53=	+	0.0000000000	0000000000	0000000000	0000000016

ATN(1/5)=	+	0.1973955598	4988075837	0049765194	7902934477
4*ATN(1/5)=	+	0.7895822393	9952303348	0199060779	1611737908

(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239)^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239)^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239)^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239)^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239)^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239)^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239)^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN(1/239)=	+	0.0041840760	0207472386	4538214959	2854527411

4*ATN(1/5)-ATN(1/239)=PI/4=	+	0.7853981633	9744830961	5660845819	8757210497

4*(4*ATN(1/5)-ATN(1/239))=PI=	+	3.1415926535	8979323846	2643383279	5028841988

The number PI correct to 40 positions after the decimal point is.

+ 3.1415926535 8979323846 2643383279 5028841971

The total number of number of terms is 35. The error of the computed value and the correct is 17 parts in the last digit, affecting the last two digits.